

# Several theorems for reflection and transmission coefficients of plane wave incidence on planar multilayer metamaterial structures

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**Abstract:** Consider an arbitrarily polarised plane wave obliquely incident on a planar multilayer structure composed of a combination of natural materials and metamaterials, then the following three new theorems are proved in this paper. *Theorem 1:* Consider a planar multilayer structure made of a combination of common materials and metamaterials situated between two half spaces composed of lossless media. Now each layer is filled by its dual media according to the interchanges  $DPS \leftrightarrow DNG$  and  $ENG \leftrightarrow MNG$ . Then, the reflection ( $R$ ) and transmission ( $T$ ) coefficients from the structure become the complex conjugates of their counterparts. Consequently, the reflected power and transmitted power from the structure are the same for the two dual cases. *Theorem 2:* If the interchanges  $DPS \leftrightarrow DNG$  and  $ENG \leftrightarrow MNG$  are made in all the layers except in the half spaces on the two sides of the multilayer structure, then the reflection coefficients become complex conjugates and the reflected power remains the same. *Theorem 3:* If a planar multilayer structure is backed by a perfect electric conductor and the media interchanges  $DPS \leftrightarrow DNG$  and  $ENG \leftrightarrow MNG$  are made in the layers, then the reflection coefficients of the two dual structures become complex conjugates of each other, and the reflected powers are equal. Finally, several examples and applications with dispersion are included.

## 1 Introduction

Metamaterials (MTMs) are categorised by negative real parts of permittivity and/or permeability [1–5]. They may be classified as epsilon negative (ENG), mu negative (MNG) and double negative (DNG). Natural materials are called double positive (DPS). Other designations are also used for them. Several numerical methods have also been developed for the computation of their various characteristics [6–8]. We consider a multilayered planar structure consisting of several homogeneous layers composed of metamaterials and common materials [9–12]. The structure may be placed between two half spaces filled with homogeneous materials (or free space), or the structure may be backed by a perfect electric conductor (PEC) plane. Assume that a plane wave with transverse magnetic (TM), transverse electric (TE), circular or elliptical polarisation is obliquely

incident at an angle of incidence on the multilayer structure, as shown in Fig. 1. Various methods have been devised for the analysis of wave incidence on multilayered structures [13–16].

In this paper, we use a full-wave matrix method for the analysis of this problem, whereby the field in each layer is decomposed into forward and backward travelling waves. Subsequently, we prove several interesting theorems for the reflection and transmission coefficients of incident plane waves with general polarisation onto the planar multilayered metamaterial structures. These theorems are independent of the angle of incidence, frequency, number of layers, their thicknesses and the type of plane wave polarisation. These theorems reveal some type of duality between DPS and DNG media and also between ENG and MNG media.