

State-transition-matrix method for inverse scattering in one-dimensional inhomogeneous mediaDavoud Zarifi,^{*} Mohammad Soleimani,[†] and Ali Abdolali[‡]*Antenna and Microwave Research Laboratory, School of Electrical Engineering,
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(Received 12 May 2014; revised manuscript received 20 August 2014; published 7 November 2014)

This study presents an analytical approach for the electromagnetic characterization of one-dimensional inhomogeneous media. The proposed approach provides the permittivity profile of the medium in terms of the reflection and transmission coefficients. The inverse solution of the permittivity profile is obtained with the help of the state-transition matrix (STM) and its properties, which are presented and proved. The advantage of using this analytic reconstruction technique is its ability to remove complexity and nonlinearity of the inverse problem. Several examples have been considered for validation of the proposed technique and, in each case, quite good agreement has been found between the original and reconstructed profiles. It has been established from the obtained results that when the scattering parameters are combined with the properties of STM, a robust and reliable technique is provided for the electromagnetic characterization of one-dimensional inhomogeneous media.

DOI: [10.1103/PhysRevE.90.053203](https://doi.org/10.1103/PhysRevE.90.053203)

PACS number(s): 41.20.Jb, 02.30.Zz, 78.20.Ci

I. INTRODUCTION

The study of electromagnetic inverse scattering is a widely encountered problem and has been a subject of extensive research. The inverse scattering of one-dimensional inhomogeneous media is of great interest due to its potential functional benefits for many applications, such as remote sensing, biomedical diagnosis, industrial tomography, nondestructive testing, military surveillance, and many others. In general, the reconstruction process involves the measurement of scattering data due to an illuminating wave. Information about the unknown permittivity profile of an object in terms of measured scattering data is obtained by using some inverse techniques. Various methods have been used to reconstruct one-dimensional permittivity profiles from electromagnetic scattering data which can be categorized into time domain and frequency domain methods [1–12]. The time domain methods always require a very narrow pulse which is difficult to radiate and to be received in practice.

A survey of the literature on inverse scattering indicates that the conventional methods basically depend on source reconstruction which leads to nonlinear equations that can only be solved using iterative and optimization algorithms [13–15]. In fact, the problem with most of these numerical and quasinumerical techniques is that they are usually computationally intensive, and sometimes it is difficult to obtain a unique and stable solution for the corresponding inverse problem. Although a number of analytical techniques have been proposed to reconstruct the permittivity profiles in terms of the inverse Fourier transform of the scattering data, most of these approaches usually assume that the scattering data are available over the whole frequency band which is difficult in practice. In addition, a large number of investigations have been carried out using the Gel'fand-Levitan-Marchenko theory [16–18]. Unfortunately, this exact approach is actually very difficult to

implement due to considerable mathematical complexity. It is mainly due to these reasons that a unique analytical approach is proposed in this paper to obtain the inverse solution of the one-dimensional inhomogeneous medium.

Briefly, for homogeneous material, one can find numerous studies in which the analytical expression between material parameters and scattering parameters was solved either directly or iteratively [19–21]. A more complicated problem compared to that of homogeneous materials is the measurement of inhomogeneous structures, which has been mainly treated by the aid of numerical and optimization techniques. In this contribution, the aim of the study is to present an analytic methodology based on the state-transition-matrix (STM) method to the characterization of inhomogeneous media. The STM method has been well described in forward scattering problems, including isotropic, anisotropic, and bianisotropic media, over the years [22–26]. Recently, its application in the formulation for inverse scattering problems, including homogeneous media, has been proposed [27,28].

Organization of this paper is as follows. The paper starts with a brief discussion of application of the STM method in inhomogeneous media. Section III deals with the proof of some useful properties of the STM of an inhomogeneous layer. The inverse algorithm based on the transition-matrix method is then explained in detail in Sec. IV. The proposed approach is validated along with example computations in Sec. V. Finally, a summary and conclusions are made in Sec. VI.

II. FORMULATION OF FORWARD SCATTERING PROBLEM

The geometry of the medium under investigation is shown in Fig. 1. A time-harmonic electromagnetic wave is normally incident from the left upon an inhomogeneous dielectric slab with permittivity ε which is a function of the geometric distance z . In the spectral domain approach, the time dependence is $e^{j\omega t}$. The planar structure is of infinite extent along the y direction, and so the derivative of the fields with respect to the y variable vanishes. In addition, the derivative of the fields with respect to the x variable in the slab must take

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