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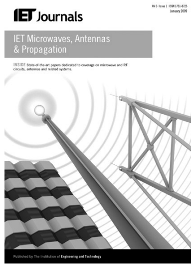

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State transition matrix of inhomogeneous planar layers

Davoud Zarifi, Mohammad Soleimani, Ali Abdolali

Antenna and Microwave Research Laboratory, Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran
E-mail: zarifi@iust.ac.ir

Abstract: In this study, the computation and properties of the state transition matrix of inhomogeneous planar layered media are investigated. Furthermore, non-reciprocity of inhomogeneous planar layered media is shown using the transition matrix method. This theorem says that the plane wave transmission coefficients for a slab with inhomogeneity along the direction perpendicular to the interfaces are the same when the wave incidences on the slab from left or from right, but that the reflection coefficients usually differ. The validation of the results is studied finally through some typical examples.

1 Introduction

Features of electromagnetic wave propagation in inhomogeneous media have been intensively studied during the last decades. Inhomogeneous media are widely used in microwave and antenna engineering as shields, filters, absorbers and radomes [1–7]. In addition, inhomogeneous media provide less scattering, larger bandwidth and better coupling effects than homogeneous media.

Although analysis of scattering from stratified inhomogeneous media is more complicated than that from homogeneous media, several approaches have been presented for the analysis of the scattering from inhomogeneous media such as Richmond method [8], Riccati equation [9], Taylor's series expansion [10] and Fourier series expansion [11]. The transition matrix method is a commonly used to deal with the problems of plane wave scattering from planar layered inhomogeneous media [12–15]. The most important feature of this method is that no matter how complex the medium is under study, the transverse components of electric and magnetic fields with some algebraic manipulating become four coupled first-order differential equations.

This paper deals with the computation of state transition matrix of inhomogeneous planar layered media using an analytic method based on the Peano–Baker series. In addition, we show non-reciprocity of one-dimensional (1D) inhomogeneous planar layered media using the transition matrix method. In the other word, it is shown that the plane wave transmission coefficients for a slab with inhomogeneity along the direction perpendicular to the interfaces are the same when the wave incidences on the slab from left or from right, but that the reflection coefficients usually differ. Notice that, here, the medium is reciprocal and non-reciprocity is used for the interaction of electromagnetic waves with the inhomogeneous slab from left and right.

2 Review on the transition matrix method

Consider an inhomogeneous slab characterised by a set of constitutive relations

$$\bar{D} = \varepsilon_0\, \varepsilon(z)\, \bar{E} \tag{1a}$$

$$\bar{B} = \mu_0\, \mu(z)\, \bar{H} \tag{1b}$$

where $\varepsilon(z)$ and $\mu(z)$ are relative permittivity and permeability, respectively. As shown in Fig. 1, the planar structure is of infinite extent along the y-direction so, derivatives of the fields vanish with respect to y and z variables, that is, $\partial/\partial y = 0$ and $\partial/\partial x = -jk_0\sin\theta_0$. By eliminating z-components of electric and magnetic fields from curl Maxwell's equations one can write

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix}\bar{E}_{\mathrm{T}}\\ \bar{H}_{\mathrm{T}}\end{bmatrix} = \mathbf{\Gamma}\begin{bmatrix}\bar{E}_{\mathrm{T}}\\ \bar{H}_{\mathrm{T}}\end{bmatrix} \tag{2}$$

where $\bar{E}_{\mathrm{T}} = (E_x, E_y)$ and $\bar{H}_{\mathrm{T}} = (H_x, H_y)$ and are transverse components of electric and magnetic fields, respectively, and $\mathbf{\Gamma}$-matrix is given by (see equation (3) on the bottom of the next page)

where ω, c and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ are the angular frequency, speed of light and the intrinsic impedance of free space, respectively.

Defining a 4×4 state transition matrix $\mathbf{\Phi}$ with 2×2 sub-matrices $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$, $\mathbf{\Phi}_3$ and $\mathbf{\Phi}_4$ that relates the transverse components of electric and magnetic fields at two