

**Parameter retrieval of chiral metamaterials based on the state-space approach**Davoud Zarifi,<sup>\*</sup> Mohammad Soleimani,<sup>†</sup> and Ali Abdolali<sup>‡</sup>*Antenna and Microwave Research Laboratory, School of Electrical Engineering,**Iran University of Science and Technology (IUST), Tehran, Iran*

(Received 27 March 2013; published 29 August 2013)

This paper deals with the introduction of an approach for the electromagnetic characterization of homogeneous chiral layers. The proposed method is based on the state-space approach and properties of a  $4 \times 4$  state transition matrix. Based on this, first, the forward problem analysis through the state-space method is reviewed and properties of the state transition matrix of a chiral layer are presented and proved as two theorems. The formulation of a proposed electromagnetic characterization method is then presented. In this method, scattering data for a linearly polarized plane wave incident normally on a homogeneous chiral slab are combined with properties of a state transition matrix and provide a powerful characterization method. The main difference with respect to other well-established retrieval procedures based on the use of the scattering parameters relies on the direct computation of the transfer matrix of the slab as opposed to the conventional calculation of the propagation constant and impedance of the modes supported by the medium. The proposed approach allows avoiding nonlinearity of the problem but requires getting enough equations to fulfill the task which was provided by considering some properties of the state transition matrix. To demonstrate the applicability and validity of the method, the constitutive parameters of two well-known dispersive chiral metamaterial structures at microwave frequencies are retrieved. The results show that the proposed method is robust and reliable.

DOI: [10.1103/PhysRevE.88.023204](https://doi.org/10.1103/PhysRevE.88.023204)

PACS number(s): 41.20.Jb, 81.05.Xj, 78.67.Pt, 78.20.Ek

**I. INTRODUCTION**

The interaction of electromagnetic waves with chiral media has long been a subject of interest. Chiral media have two important properties: namely optical activity which may rotate the polarization plane of a linearly polarized wave propagating through it, and circular dichroism which is attributed to the different absorptivity of right- and left-circularly polarized waves inside the chiral medium [1]. Recently, more and more interest has been focused on the chiral metamaterials (CMMs) due to their attractive properties such as strong optical activity and negative refractive index in microwave and optics. CMMs are metamaterials (MTMs) made of unit cells without any mirror symmetry. This property results in the breaking degeneracy of two circularly polarized waves, i.e., right- and left-circularly polarized waves with different refractive indices. Negative refraction can be realized in CMMs, with neither negative permittivity nor permeability required. Based on this concept, several semiplanar structures with fourfold rotational symmetry have been suggested as the different kinds of CMMs [2–13]. In addition, numerous applications have been proposed for chiral metamaterials such as realizing structures with strong optical activity [14–16], realizing negative refraction index in the terahertz regime [17,18], circular polarizer [19,20], wide angle and polarization independent microwave absorber [21], and gain enhancement and axial ratio improvement of circularly polarized antennas [22].

Retrieving the effective medium parameters of MTMs is a critical concept in MTMs design and can be achieved by several approaches of which the  $S$ -parameters method is the most prevalent one [23–25]. This approach has been modified in [26] in order to extract effective parameters of CMMs by considering a CMM slab with thickness  $d$  illuminated by normal incidence of circularly polarized waves. Recently, an improved algorithm was derived based on Kramers-Kronig relations for the unique extraction of effective parameters of CMMs [27]. Briefly, in the standard full-wave parameter retrieval methods, the eigenwaves of the wave equation in an isotropic chiral layer are derived and the general form of electric and magnetic fields inside such an environment are determined. Then, using the boundary conditions of the problem, the constitutive parameters of the CMM structure at normal incidence are retrieved.

The objective of the present work is to present an alternative parameter retrieval method for the electromagnetic characterization of chiral slabs using the state transition matrix method. The transition matrix method is commonly used to deal with the problems of plane wave scattering from planar layered generalized anisotropic or bianisotropic media, and its application in forward scattering problems has been well studied over the years [28–33]. In this work we apply the formulation to inverse scattering problems. The proposed retrieval method which uses scattering data corresponding to linearly polarized waves is mainly based on the properties of the state transition matrix of an isotropic chiral layer which are discussed as two theorems.

The paper is organized as follows. Section II is dedicated to review of the forward problem analysis using the state transition matrix method. Two properties of the state transition matrix of a chiral layer are presented in Sec. III. Section IV deals with the detailed formulation of parameter retrieval

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